**NOISE FUNDAMENTALS: JOHNSON NOISE**

Last Revision: February 23, 2015

QUESTION TO BE INVESTIGATED

What is Johnson noise? How does it depend on resistance, frequency, and temperature?

INTRODUCTION

*Much of this lab manual is composed of adaptations of selected portions of the instructor’s manual for this equipment* [1]*.*

What is noise? A noise, *ordinarily speaking*, is an unwanted sound. In fields such as physics or electrical engineering, we extend the definition of 'noise' beyond acoustics to the general field of information. Since almost any signal that's a function of time can be translated into a voltage, we will often use the concept of a voltage signal. We'll call it a 'noisy signal' if, in addition to the voltage we expect or wish to see, there is unwanted, typically randomly-fluctuating, voltage. Surprisingly, the noise signal is sometimes not only wanted, but is the essence of the measurement.

There are several kinds of noise. One of them is 'interference', which is the presence of an unwanted signal, added to the desired signal. It's easy to imagine that your neighbor's electronic apparatus is polluting your TV or radio signal with some sort of interference. The kind of interference students are likely to encounter in these experiments probably comes from three sources: electrostatic coupling to the apparatus from fluorescent lights in the laboratory, electromagnetic coupling due to nearby transformers or motors, and vibrational coupling due to microphonic components within the unit.

Another source of noise we will call 'technical noise' since it is the noise generated by the technique of the investigation, or that gets into the circuits due to *faulty* experimental techniques. For example, failure to tighten the cover on the preamplifier section, or a poor electrical connection to the first-stage op-amp, can add extraneous noise to the signal path.

Of greatest interest to us is 'fundamental noise', noise that is intrinsic and inevitable because of the physical nature of an apparatus. Using this equipment, it is possible to observe noise sources that arise from the Second Law of Thermodynamics, and from the quantization of electrical charge. Physicists and electrical engineers know these as Johnson and shot noise respectively. In this experiment, you will only work with Johnson noise[[1]](#footnote-1). Noise sources like this display the characteristics of non-periodic, unpredictable, random waveforms, but nevertheless conforming, in their statistical properties, to universal laws.

Fundamental noise is especially worthy of study, for at least two reasons. The first reason is that fundamental noise presents us with a physics-based limit on the degree to which we can measure in a given experiment. In many cases in research and technology, it often defines what is possible within the limits of physical law. In particular, fundamental noise can and does set limits to the rate of data-transfer in a host of contexts in communication.

The second reason we care about noise is that it becomes possible to use noise to measure the values of some fundamental constants. Boltzmann's constant *k*B can be determined from the voltage or Johnson noise of resistors; and the magnitude of the charge on the electron, *e*, can be determined from the current or shot noise of a photocurrent.

Every student knows *V* = *I R*, which says that there's a potential difference Δ*V* across any resistor *R* which has a current *I* passing through it. This of course predicts a Δ*V* of *zero* for a resistor with no current. But for deep reasons, any actual resistor at any temperature above absolute zero, will display a 'noise voltage' *VJ*(*t*) across its terminals, a potential difference that has all the character of an internal *emf* built into the resistor. The *emf* which the resistor generates is called 'Johnson noise', and it arises because of the deep thermodynamic connection between dissipation (which any resistor surely has) and fluctuations (which here show up as a fluctuating *emf*). The size of this *emf* is also predicted by fundamental theory, and it should not surprise you to learn that *VJ*(*t*) is, *on average*, zero. But *VJ*(*t*) exhibits fluctuations, positive and negative, about that average value of zero. To quantify these, we form the (always-positive) *square* of *VJ*(*t*), and time-average that, giving a 'mean square' voltage which we denote as <*V*J 2(*t*)> . The predicted value for <*V*J 2(*t*)> was first deduced by Nyquist, following Johnson's empirical discovery of the noise, and it's given by the expression



Here *kB* is Boltzmann's constant, *T* is the absolute temperature of the resistor, and *Δf*  is the frequency bandwidth used in the measurement electronics. The involvement of bandwidth Δ*f* is a first hint that 'noise' is quite distinct from 'signal'. Everyone starts with 'd.c. signals', which have nothing but a sign and a value, in Volts. Then there are 'a.c. signals', which have a magnitude (perhaps specified by amplitude, or rms value, or peak-to-peak excursion) but also a *frequency*, or a mixture of frequencies. But it is the essence of fundamental noise that it contains, or is composed of, *all frequencies*. In fact, the amount of energy we can get out of a 'noise source' depends on the *range* of frequencies to which we arrange to be sensitive, and this is the reason for the inclusion of the bandwidth-factor Δ*f* in the expression above.

We often take the square root of this mean-square noise voltage, to give a 'root-mean-square' or 'rms' measure of the noise voltage



So if we have a room-temperature resistor connected to an ideal voltmeter, and if that voltmeter responds to all frequencies under some upper frequency, then the voltmeter's instantaneous reading will *not* be zero volts, but instead will fluctuate around zero. We further assert that this is an actual *emf* intrinsic to the resistor, and it will still be present, though typically unwanted, in addition to any *IR*-drop that the resistor may exhibit. It follows that measurement of any *IR*-drop to microVolt precision in such a case would require thinking about this effect.

PROCEDURE

There is no power button for this equipment. If you plug the high level electronics (HLE) box into a wall outlet, and connect the low level electronics (LLE) box to the HLE box, both boxes should have lit LEDs indicating that the system has power. While the instructions that follow may be sufficient to complete the experiment, feel free to consult the instructor’s manual [1] (in the red binder) as needed.

You are encouraged to make as many measurements as possible in each of the three sections that follow (Johnson Noise Dependence on Resistance, Johnson Noise Dependence on Bandwidth, and Johnson Noise Dependence on Temperature). You should expect to spend about 1 day on each section. Make sure that you leave the full third lab day for the temperature dependence measurements, as it takes some time for your temperatures and voltage readings to stabilize.

*Johnson Noise Dependence on Resistance*

Set the switch to select a 'source resistor' of *R*in = 100 kΩ in the pre-amplifier module installed in the LLE box. This resistor is connected only to the high-impedance input of the first stage of amplification in the pre-amp. That first stage is wired to give a gain of 6.00, *provided* you set the feedback resistor, *R*f, to its 1-kΩ setting. (The feedback capacitance *C*f is not connected in the default mode, so its setting is irrelevant.) Read the graphics on the panel of the pre-amp to see that there is an additional amplification stage, with gain 100 following this first stage. Now you can connect the pre-amp's output, by a coaxial cable, to an oscilloscope, to see if there is any signal present. Use a rather sensitive vertical scale on your scope (of perhaps 10 mV/division sensitivity), a sweep speed of 5 µs/div on the horizontal axis, and trigger near zero volts.

Figure 1: Cabling for first use of HLE.

Left Filter: 0.1 kHz, AC

Right Filter: 100 kHz, AC

Gain Fine Adjust 30, toggle x1, toggle x10



The signals you've seen emerging from the pre-amp are rather small. So next use a BNC cable to convey the pre-amp output to the HLE box instead, where you can filter and amplify the still-small noise signals. By using the settings and cablings shown in Figure 1, you will have Johnson noise pre-amplified by factor 600 and filtered to pass only the 0.1 - 100 kHz frequency band. The additional cabling to the gain module gives an additional gain of 300. By this point you should have a noise signal on the scope that is large enough to be viewed with a vertical setting of 2V/div. To get a first, qualitative, indication that this noise signal has something to do with the original source resistor at the front end of this pre-amp/filter/main-amp chain, go back to the pre-amp, and change the source resistor from 100 kΩ to 10 kΩ. You should see the size of the noise signal on your scope decrease by about a factor of 3.

If your signals differ dramatically from those described here, check to make sure that you have cabled everything correctly and that you are using the same settings given. If you are still having difficulties, consult Appendices A.6 and A.5 in the instructor’s manual [1].

You should now have a rapidly-fluctuating signal on the scope which we claim is due mostly to Johnson noise, and which you now want to quantify. Use the cabling and settings shown in Figure 2 to configure the Multiplier module as a squarer. The multiplier circuit delivers at the MONITOR point, a real-time output voltage

 .

Notice that the multiplier output voltage (from MONITOR) is always positive, unlike the input voltage (the output of the Gain module).



Figure 1: Cabling to use the multiplier as a squarer.

Left Filter: 0.1 kHz, AC

Right Filter: 100 kHz, AC

Gain: 400, AC

Multiplier AxA, AC

To persuade yourself that the squarer is working, use the XY-display capability on your scope which can be accessed through the Display menu if you are using a Tektronix scope. If everything is connected as in Figure 2, you should see a parabola emerge. See to it that you understand the origin of your XY-coordinate system, and then try changing some things: What are the right sensitivities to choose on the two axes? What would happen to your parabola if you raised the gain in the main-amplifier module of the HLE? Why does your data lie on a parabola, after all?

Now withoutthe need for a further cable, the output of the squarer is already being sent internally to the Meter module of your HLE. What this module does is to take the time average of *V*out(*t*), averaged over a time interval you can select (by switch) to 1.0 second. This time average will *not* be zero, since *V*out(*t*), though fluctuating, is always and only on the *positive* side of zero. Use the 0-2 V scale, and when making any measurements, always change the main-amp gain so that the meter is near the middle of the scale, close to 1 V. This will help to avoid a clipped signal. As you may have to change the gain several times, make sure that you always record your net gain with each measurement.

What can you infer from this? Start with *V*J(*t*), the actual instantaneous Johnson-noise voltage generated by the source resistor. At the output of the pre-amp, you have a signal: (6.00)(100) *V*J(*t*). After the filter stages, you have the 0.1-100 kHz bandwidth-selected, or filtered, part of this signal. After the main amp, you have a signal *G*2(600)*V*J(*t*), where *G*2 is the main-amp gain, perhaps 400. Then after the squarer, you have a signal [(400)(600)*V*J(*t*)]2 / (10 V) . Finally, using the <...> brackets to indicate a time average, what you have displayed on your meter is the signal *V*meter = <*V*J 2(*t*)> [(400)(300)]2/(10 V) . From this result and the meter reading, you can work all the way backwards to find <*V*J2(*t*)>, the mean-square voltage present (within your chosen bandwidth) across the source resistor.

Rather than just trying to get a reading off of the dial meter, connect the output of the meter to a digital multimeter. The number should be consistent with the dial meter, and it will fluctuate. You should notice that the multimeter reading fluctuates around a mean value. Take several readings, about one every second or so, and use the average.

While it may seem that you can now obtain the Johnson noise from your resistor, there is one more thing to account for. Some of the signal you are now seeing is actually from the amplifier chain you are using, but there is a way to separate the Johnson noise from the amplifier noise. Let VJ(t) be the Johnson noise from the source resistor, VN(t) be the amplifier noise, and G be the gain of the amplifier. The output of the amplifier is then

 .

Notice if you square this output, you would have a cross term 2*V*J(*t*)*V*N(*t*), but you can assume that the *time average* of this term is zero. Since the sources of these voltages are uncorrelated, when one of them is positive, the other is just as likely to be negative or positive, meaning that the product is equally likely to be positive or negative. That's why the absence of correlation enforces a zero for the time average of the product. But that means that:

.

In other words, the mean-square voltages from uncorrelated sources are simply additive. We can get a reasonably good measure of this amplifier noise by using a configuration in which the Johnson noise term is negligible, such as the *R*=1Ω setting (Theory says that <VJ2(t)>=0 for *R*=0, but 1Ω is close enough).

Using Equations 3 and 5, we get that the time averaged output of the squarer is



where G1=600 and G2 is the product of gains selected by you n the HLE. So for the same configuration of the amplifier, you can use Equation 6 to determine the amplifier noise (by assuming that the Johnson noise is negligible at *R*=1Ω), and then inserting this value of the amplifier noise to determine the Johnson noise for other resistor values.

Now that you know how to subtract out the amplifier noise, you are ready to investigate Johnson noise dependence and resistance. Using the methods described above and Equation 6, determine the mean-square Johnson noise, <VJ2(t)> for the source resistors R = 10 Ω through 1 MΩ built into the pre-amp module (The tolerance of these resistors is 0.1%.1 ) Your data should look linear, at least for the lower resistances. There is a capacitive effect at the input of the pre-amp that is most noticeable at higher resistances. See Section 2.2 and Appendix A.8 in the instructor’s manual [1] for a more detailed explanation.

*Johnson Noise Dependence on Bandwidth*

You will now investigate how the choice of bandwidth matters. Start with a value of Rin = 10 kΩ. You have a range of choices for the 'lower corner' frequency f1 or high-pass filter setting, and a separate range of choices for the 'upper corner' frequency *f*2 or low-pass filter setting. You might first think that the bandwidth Δ*f* should be given by |*f*2 - *f*1|, which is a decent approximation, but subject to significant corrections. Corrections which are the result of a model calculation are presented here in Table 1. For more details see Section 2.2 of the instructor’s manual [1].



Table 1: Effective bandwidths for given values of *f*1 and *f*2, subject to uncertainties of order 4%.  
TaLeft Filter: 0.1 kHz, AC

Right Filter: 100 kHz, AC

Gain: 400, AC

Multiplier AxA, AC

Using the methods of the previous section, measure the mean-square Johnson noise of the resistor, <VJ2(t)>, for as many different (*f*1, *f*2) combinations as time allows. If time allows, you should also repeat your measurements for another resistance value, such as 100kΩ. Recall that for each choice of filter settings, you'll want to adjust the gain so as to use the squarer optimally. Recall that each mean square value you measure needs to be corrected for amplifier noise (measured at that bandwidth setting: the amplifier-noise contribution to the mean-square depends, as does the Johnson-noise contribution, on the bandwidth you use.)

*Johnson Noise Dependence on Temperature*

In this section, you will investigate the Johnson noise dependence on temperature. A general procedure is presented here, but it is likely that you will need to consult Chapter 4 of the instructor’s manual [1] at some point. In the end, you will need three values of <VJ2(t)>: one at the boiling point of liquid nitrogen (77K), another at room temperature, and a third point at approximately 350 K. Make sure that you wait sufficiently long at each temperature to ensure that your temperatures and voltages have stabilized.

**SAFETY WARNING:** The Dewar supplied is made of un-silvered glass to help you see the liquid level inside. Because it's made of glass, it will shatter if you drop it. The disaster will be even more dangerous if the Dewar is full of LN2 when dropped. So: *do* ***NOT*** drop the Dewar, and use and store it only in the base built to hold it securely.

**SAFETY WARNING:** Liquid nitrogen is *very* cold, boiling at about -195 °C. It is dangerous to have it contact your skin, and even more dangerous to undergo skin contact with clothing soaked with LN2. The hazard is not chemical, but physical. You can suffer frostbite, and permanent nerve and/or tissue damage, from the localized freezing that will occur.

You will start by taking data at room temperature. Connect the probe cable to the thermal module, and make sure that the thumbscrews attaching the probe to the mounting are hand-tight. Now you should be able to measure (room-temperature) Johnson noise from the three remote resistors inside the probe, just by using the A, B, C, positions of the source-selector (Rin) switch on the pre-amp. The resistance values are as follows: *R*A = 10 , *R*B = 10 k, and *R*C = 100 k(all with a 1% tolerance).

For initial measurements, we suggest a bandwidth of about 10 kHz (set perhaps by using a 1-kHz high-pass, and a 10 kHz low-pass, filter). As usual, you'll need to recall that the standard gain is *G*1 = 600 in the pre-amp, and you'll need to use a suitable gain *G*2 in the main-amp to get the squarer to operate in its optimal regime. As previously, here too you'll need to use the 10source resistor as a way to get the amplifier-noise contribution, which needs to be subtracted from the mean-square noise measurements.

Take data from both of the 10 kand 100 kremotesource resistors, and also try some different bandwidths. Because of the effects of probe capacitance, it is to be expected that the values of <*V*J2(*t*)> you infer will be smaller for the remote, as compared to the local, resistors.

Use these (all room-temperature) results to decide on a measurement strategy that you'll use when the probe is *not* at room temperature. When you've worked that out, it is finally time to cool your probe. With the dewar mounted in its movable base, lower the dewar and base and pour about 1 liter of LN2 into the dewar (Wear the gloves and safety glasses provided. If you are not comfortable doing this yourself, ask your instructor, TA, or the laboratory coordinator to do it for you.) Wait for the boiling to subside, slide the black foam insulating cover down onto the Dewar's mouth, and now use the clamp on the Dewar's base to raise the Dewar until the probe makes contact with the LN2. Here the probe's sample chamber should end up at about the mid-height in the Dewar with its copper bottom plate immersed in the liquid.

When all the extra boiling has settled down, you can repeat your noise measurements using the protocol you've established. Do this for both the10 kand 100 kresistors. You may need to change the gain G2 to keep the squarer in its optimal regime. When you are done, carefully lower the dewar and base. Pour any excess liquid nitrogen back into the large 10 L dewar. (Once again, if you are uncomfortable doing this, ask the instructor/TA/lab coordinator for help.) Put the small dewar back on the base.

Now that you have values of <*V*J2(*t*)> for both room temperature and submersed in liquid nitrogen, you will use the built-in heater to measure the Johnson noise at a higher temperature (~350 K). Unfortunately, the heater does not have a direct temperature readout. You will have to use Table 2 to convert voltage measurements into temperatures. In order to use Table 2, you must set the Current Source on the Temperature Module to 10 µA. You can measure the voltages by connecting a multimeter to the Monitor output of the Temperature Module. The data in the table is approximately linear, so one possible way to make use of it is to plot your results and obtain a linear fit (Is your linear fit perfect? Just something to think about for error analysis). Turn the Heater Voltage knob on the Temperature module to about 5 (half of full power). Wait until the temperature (i.e., the voltage reading from the Monitor output of this module) stabilizes. It may take some time, especially if the probe is still well below room temperature due to the liquid nitrogen measurements. Once you are convinced that the temperature is relatively stable, adjust the Heater Voltage knob to obtain a probe temperature inside the range of 350±10 K. Once the temperature and voltage from the squarer have stabilized, determine <*V*J2(*t*)> at this temperature for both the10 kand 100 kresistors.

When you are sure that you are done taking data, turn the Heater Voltage knob all the way back down to zero, and unplug the HLE unit from the wall.

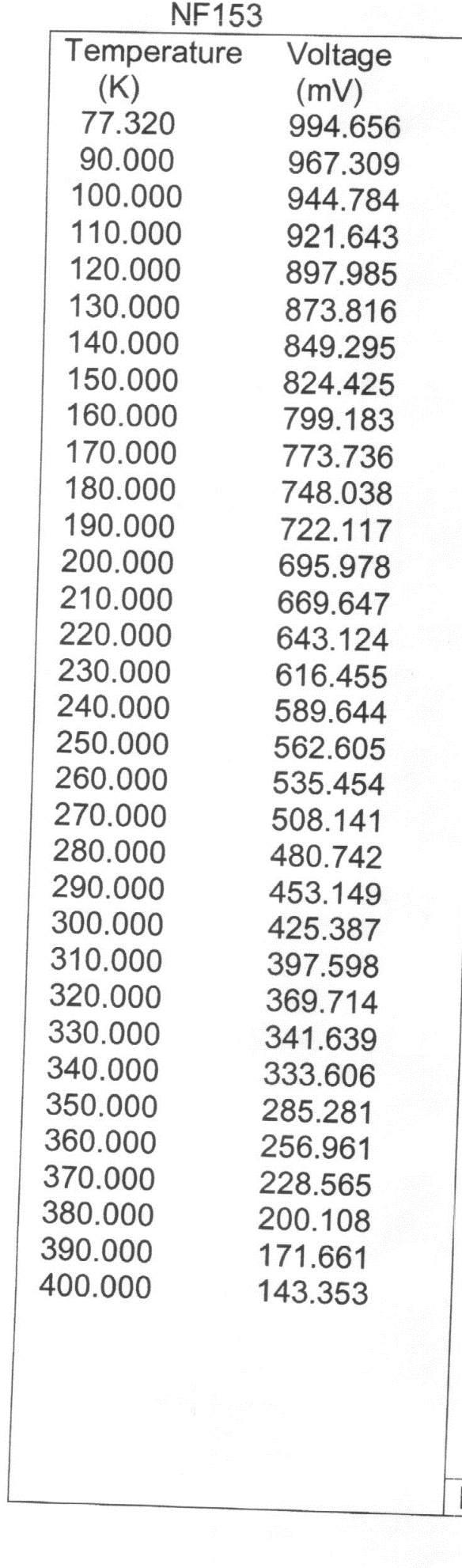


Table 2: Table for converting voltage readings from the Monitor output of the Temperature module into temperatures. The current must be set to 10 µA.

ANALYSIS

For all of your results, discuss your sources of error, both qualitatively and quantitatively when possible. You may find it helpful to read Chapter 2 in the instructor’s manual [1] before delving into your analysis. For each of the sections of your experiment, at a minimum, make sure to include the following:

*Johnson Noise Dependence on Resistance*

Plot your amplifier noise corrected values of <VJ2(t)> as a function of *R*. Create a log-log plot and determine the slope. What does Nyquist’s theory (Equation 1) predict? Is your data consistent with the theory? Are there certain resistance ranges where your data is not consistent with Equation 1? Discuss. (It is recommended that you read Section 2.2 and Appendix A8 of the instructor’s manual if you have not yet done so.)

*Johnson Noise Dependence on Bandwidth*

Plot your data for <VJ2(t)> as a function of both |*f*2 - *f*1| and the equivalent bandwidth from Table 1. Which is more nearly linear? Again, create a log-log plot as well. Are there any ranges where your data is not (almost) linear? In the previous section, you should have determined a regime where <VJ2(t)> depends linearly on *R*. For one of the resistances in this regime, obtain a value for Boltzmann’s constant, kB, using the slope of your <VJ2(t)> vs. Δ*f*  plot (NOT the log-log plot). What uncertainty can you assign to your value?

*Johnson Noise Dependence on Temperature*

Plot your data for <VJ2(t)> as a function of temperature for both the10 kand 100 kresistors. Again, create a log-log plot as well. Are they both linear as suggested by Equation 1? Discuss. If you had more time on this experiment, how could you improve your investigation of temperature dependence?

# REFERENCES

|  |  |
| --- | --- |
| [1] | TeachSpin, Inc. and David Van Baak, Noise Fundamentals NF1-A Instructor's Manual, 2010. |

1. Advanced students may wish to investigate shot noise as well. If you want to work with shot noise, please consult with both your instructor and the laboratory coordinator, as this may require some reconfiguration of the lower level electronics. A presentation of shot noise is not presented here, but can be found in Chapter 3 of the instructor’s manual. If you have reconfigured any of the lower level electronics, you must return it to the default configuration as shown in Section 7.2 of the instructor’s manual. [1] [↑](#footnote-ref-1)